Further comment on pion electroproduction and the axial form factor

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We show that a recent claim [1] that one cannot extract the nucleon weak axial form factor $G_A(t)$ from charged pion threshold electroproduction is incorrect. Thus previous calculations remain valid and threshold charged pion electroproduction experiments can indeed be used to determine $G_A(t)$, and they should certainly be pursued.

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In a recent paper, Haberzettl [1] claims that one cannot extract the weak axial form factor $G_A(t)$ of the nucleon from threshold electroproduction of charged pions as first stressed by Nambu and collaborators [2]. He argues that previous derivations of the relationship between $G_A(t)$ and the electromagnetic structure of the Kroll-Ruderman contact terms are based on incomplete evaluations of the relevant PCAC expressions and that, if all mechanisms are taken into account, the dependence of the pion electroproduction amplitude on $G_A(t)$ vanishes.

Let us follow the derivation of [1] and show were it goes wrong. (We will later discuss the method underlying that paper.) The matrix element of the weak axial current between nucleon states is indeed given by:

$$j_A^\mu = -\bar u_f(p')\gamma_5\left(\gamma^\mu G_A(t) + \frac{(p-p')^\mu}{2m}G_p(t)\right)\frac{\tau}{2}u_i(p) \ , \tag{1}$$

with $t = (p'-p)^2$ the invariant momentum transfer squared, m denotes the nucleon mass, $G_A(t)$ and $G_P(t)$ are the axial and the induced pseudoscalar form factor, respectively. Exploiting the chiral Ward identity of QCD relating the divergence of the axial current to the pseudoscalar density one gets to leading order for G_P

$$G_p(t) = \frac{4mg_{\pi NN}F_\pi}{M_\pi^2 - t} + \mathcal{O}(t^0) \ , \tag{2}$$

which is the well known leading pion pole contribution to G_p expressed in terms of the pion–nucleon coupling constant $g_{\pi NN}$ and the weak pion decay constant F_π. The corrections to this result have also been obtained but are of no interest for the following discussion. Consequently, the nucleon matrix element of the axial current, Eq.(1), thus contains a pion pole dominated part. This is a direct consequence of the spontaneous chiral symmetry breaking of QCD as first observed by Nambu. This axial current matrix element is unambiguous and unique, and so is the axial current. The key observation is now that there is no need, as has been done in [1], to split the axial current into two pieces and introduce a so–called "conserved weak part" $\hat j_{A,W}$ at the prize of introducing some unphysical and problematic singularity at $t=0$. This splitting leads to the wrong claim in [1]. This observation was already made by Guichon [3]. The axial current can indeed be represented as in fig.1 of [1], i.e. by a pion–pole term and remainder, but then $\hat j_{A,W}$ is an unconserved quantity which contains all but the pion-pole contribution and which together with $\hat j_{A,H}$ *does not contain any unphysical singularity.* This is particularly important since now $\hat j_{A,H}$ describes the creation of a pion of mass M_π out of the vacuum, with the coupling operator $-F_\pi(p'-p)^\mu$ and associated normalized "form factor" 1 and *not* M_π^2/t, and the subsequent propagation of the pion and its final absorption in the nucleon, where we have used the same phrasing as in [1]. This is nothing but the well known QCD relation: $\langle 0|A_\mu|\pi\rangle \sim F_\pi p_\mu$ which is easily recovered in chiral perturbation theory. At that point it is fairly easy to see that in the derivation of eq.(19) of ref. [1], which is the main point of Haberzettl's note, the divergence of the last term in eq.(14) of that paper which is proportional to $\hat j_\pi^\mu$ will *not* contribute to the photoproduction amplitude $\mathcal{M}$ as claimed by Haberzettl. Indeed it does not lead to a term of the form

$$\sim \frac{M_\pi^2}{q^2 - M_\pi^2}M_{\rm int}^\nu \ , \tag{3}$$

but rather to a structure of the type

$$\sim \frac{q^2}{q^2 - M_\pi^2}M_{\rm int}^\nu \ . \tag{4}$$

where $M_{\rm int}^\nu$ is the interaction current defined in eq.(12) of [1] (see also fig.2 of [1]). The difference in these last two expressions can be traced back to the "normalized form factor" of $\hat j_\pi^\mu$. It will thus contribute to the last term $\bar u_f \mathcal{W}^\nu u_i\epsilon_\nu$ which vanishes in the soft pion limit. Thus $q_\mu J^{\mu\nu}_{A,\gamma}\epsilon_\nu$ will *not* involve this contribution from $\hat j_\pi^\mu$ contrary to what Haberzettl claims. It will *indeed vanish even when* $q\to 0$.* Thus there is one term less in the

*The third term and fifth term in eq.(14) of [1] which also depend on $\hat j_\pi^\mu$ will only lead to a modified expression for $\mathcal{W}^\nu$.

photoproduction amplitude, the one corresponding to the last diagram in fig.2 of [1]. We have:

$$q_\mu J^{\mu,\nu}_{A,\gamma}\epsilon_\nu = \frac{f_\pi M_\pi^2}{q^2 - M_\pi^2}(\mathcal{M} - \mathcal{M}_{\rm int}) + \bar{u}_f \mathcal{W}^\nu u_i \epsilon_\nu \tag{5}$$

where $\mathcal{M}_{\rm int}$ contains the Kroll-Ruderman contact term among others. But as shown by Haberzettl, this contact term is just given by $Q_\pi j_A^\nu \epsilon_\nu$ in the soft pion limit, so that one naturally gets back to eq.(19) of [1] as it should. Note that $\mathcal{W}^\nu$ does not verify eq.(20) anymore, the term $Q_\pi j^\nu_{A,W}$ is replaced by

$$\frac{q^\nu}{t - M_\pi^2}\,\gamma_5\, g_{\pi NN}\,\tau \tag{6}$$

and one has additional terms coming from other diagrams contributing to $\mathcal{M}_{\rm int}$. The important point is that now the axial form factor $G_A(t)$ enters only via $Q_\pi j_A^\nu \epsilon_\nu$ in the soft pion limit as in all previous calculations and contrary to Haberzettl's claim. There is no more cancellation of the axial form factor which was solely coming from this ad hoc splitting of the axial current into parts containing unphysical singularities at $t = 0$.

We also remark that the method of [1] applied to this particular problem is extremely clumsy. The correct relation based on the chiral Ward identities of QCD can be obtained much more easily by making use of an effective Lagrangian as it is known since decades.† Indeed, we have used such methods to derive the one–loop corrections to the NLS [2] low-energy theorem, which is claimed to be erroneous by Haberzettl, in [4]. The steps in [4] are extremely simple to follow and they also show that Haberzettl's metaphysical remarks about the relation of his results to the ones obtain in chiral perturbation theory are unfounded. Current algebra is nothing but the first term in a systematic expansion about the chiral limit of QCD and thus must lead to the same result as a corresponding lowest order chiral perturbation theory calculation. Of course, one has to be aware of possible pitfalls - some current commutators are not well defined and making extra assumptions can lead to incorrect results, the best example being the incorrect low–energy theorem for neutral pion photoproduction off nucleons [5]. In fact, the NLS low–energy theorem was exactly missing the additional "axial radius correction" found in [4] because the smoothness assumption of going from massless to massive pions does not commute with taking the derivative of the electric dipole amplitude $E^-_{0+}(k^2)$ with respect to the photon virtuality k^2 for k^2 tending to zero. Needless to say that using the method of [1] it seems impossible to us to recover such an intricate correction. Independent of this correction, the effective Lagrangian method directly leads to the NLS result and the author of [1] has to proof that that derivation is also incorrect. Parenthetically, we are utterly amazed that the referees of [1] did not even perform this extremely simple check.

We have thus shown that the claim of Haberzettl that one cannot extract the nucleon weak axial form factor $G_A(t)$ from threshold pion electroproduction in fact is wrong. It is vital to perform further precise charged pion electroproduction experiments to not only get a better determination of the nucelon axial form factors but also have an alternative method to measure the pion charge radius, see e.g. [6].

†To quote a famous Harvard physicist, the method of [1] appears to us as an exercise in self-torture.

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